(1) Find Laplace Inverse of —

$$F(s) = \frac{7}{s^2+4}$$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{7}{s^2+4}\right\}$$

or, $f(t) = 7\,L^{-1}\left\{\frac{1}{s^2+4}\right\}$

$$L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin(at)$$

or, $L\{\sin at\} = \frac{a}{s^2+a^2}$

or, $f(t) = \frac{7}{2}\,L^{-1}\left\{\frac{2}{s^2+2^2}\right\}$

or, $\boxed{f(t) = \frac{7}{2}\sin(2t)}$

(2) $F(s) = \frac{5}{s-4} + \frac{7s}{s^2+16}$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{5}{s-4} + \frac{7s}{s^2+16}\right\}$$

$$L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

or $f(t) = 5\,L^{-1}\left\{\frac{1}{s-4}\right\} + 7\,L^{-1}\left\{\frac{s}{s^2+16}\right\}$

or $\boxed{f(t) = 5e^{4t} + 7\cos(4t)}$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos(at)$$

(3)

$$L^{-1}\left\{\frac{s+1}{s^2+4s-5}\right\}$$

<u>Partial fraction.</u>

$$\frac{s+1}{s^2+4s-5} = \frac{(s+1)}{(s^2+4s+4-9)}$$

$$= \frac{(s+1)}{(s+2)^2-9}$$

$$= \frac{(s+2)-1}{(s+2)^2-9}$$

$$= L^{-1}\left\{\frac{(s+2)}{(s+2)^2-9} - \frac{1}{(s+2)^2-9}\right\}$$

$$= L^{-1}\left\{\frac{(s+2)}{(s+2)^2-3^2}\right\} - L^{-1}\left\{\frac{1}{(s+2)^2-9}\right\}$$

By shifting Property -

if

$L\{f(t)\} = F(s)$

$L\{e^{at}f(t)\} = F(s-a)$

i.e.

$L^{-1}\{F(s-a)\} = e^{at}L^{-1}\{F(s)\}$

$= e^{at}f(t).$

$$= e^{-2t}L^{-1}\left\{\frac{s}{s^2-3^2}\right\} - e^{-2t}L^{-1}\left\{\frac{1}{s^2-9}\right\}$$

$$= e^{-2t}\cosh(3t) - \frac{e^{-2t}}{3}L^{-1}\left\{\frac{3}{s^2-9}\right\}$$

$$= e^{-2t}\cosh(3t) - \frac{e^{-2t}}{3}\sinh(3t)$$

$$= \frac{e^{-2t}}{3}\left[3\cosh(3t) - \sinh(3t)\right]$$

and,

$L\{\cosh(at)\} = \frac{s}{s^2-a^2}$

$L\{\sinh(at)\} = \frac{a}{s^2-a^2}.$

so,

$$\boxed{L^{-1}\left\{\frac{s+1}{s^2+4s-5}\right\} = \frac{e^{-2t}}{3}\left[3\cosh(3t) - \sinh(3t)\right]}$$

(4) $F(s) = \dfrac{s}{(s-4)^5}$

or, $F(s) = \dfrac{(s-4+4)}{(s-4)^5}$

or $F(s) = \dfrac{(s-4)}{(s-4)^5} + \dfrac{4}{(s-4)^5}$

or $F(s) = \dfrac{1}{(s-4)^4} + \dfrac{4}{(s-4)^5}$

∴ By shifting Property:
If $L^{-1}\{F(s)\} = f(t)$
then, $L^{-1}\{F(s-a)\} = e^{at} f(t)$.

Taking Laplace inverse -

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{(s-4)^4}\right\} + 4 L^{-1}\left\{\frac{1}{(s-4)^5}\right\}$$

or $F(t) = e^{4t} L^{-1}\left\{\dfrac{1}{s^4}\right\} + 4e^{4t} L^{-1}\left\{\dfrac{1}{s^5}\right\}$

$L^{-1}\left\{\dfrac{1}{s^n}\right\} = \dfrac{t^{n-1}}{(n-1)!}$

or $F(t) = e^{4t} \cdot \dfrac{t^3}{\lfloor 3} + 4e^{4t} \cdot \dfrac{t^4}{\lfloor 4}$

or, $F(t) = \dfrac{e^{4t}}{6}\left[t^3 + t^4\right]$

$$\boxed{\text{or, } F(t) = \frac{t^3 e^{4t}}{6}(t+1)}$$

(5) $$F(s) = \frac{e^{-3s} - e^{-6s}}{s^8}$$

or, $$F(s) = \frac{e^{-3s}}{s^8} - \frac{e^{-6s}}{s^8}$$

Taking Laplace Inverse-

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{e^{-3s}}{s^8}\right\} - L^{-1}\left\{\frac{e^{-6s}}{s^8}\right\}$$

Now,

$$L^{-1}\left\{\frac{1}{s^8}\right\} = \frac{t^7}{7!}$$

Thus,

$$L^{-1}\left\{\frac{e^{-3s}}{s^8}\right\} = \frac{(t-3)^7}{7!} U(t-3)$$

and,

$$L^{-1}\left\{\frac{e^{-6s}}{s^8}\right\} = \frac{(t-6)^7}{7!} U(t-6)$$

$$L\{f(t)\} = F(s)$$

$$L\{f(t) \cdot U(t-a)\}$$

$$= F(s-a) \cdot e^{-as}$$

So, inverse-

$$L^{-1}\{F(s)\} = f(t)$$

$$L^{-1}\{e^{-as} F(s)\} = f(t-a) \cdot U(t-a)$$

$$\boxed{U(t-a) = \begin{cases} 1 & 0<t<a \\ 0 & t>a \end{cases}}$$

Unit step function.

and,

$$L^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$$

so,

$$\boxed{f(t) = L^{-1}\{F(s)\} = \frac{1}{7!}\left[(t-3)^7 \cdot U(t-3) - (t-6)^7 U(t-6)\right]}$$